Deposit and Interest Rate Risk: Some Simple But Surprising Results

Bill Nelson | Jan. 29, 2024

The bank failures in spring 2023 highlighted the risk of a bank relying on its deposit franchise as a hedge for the interest rate risk posed by holding longer-term assets: the hedge is rendered ineffective if depositors become so concerned about the bank's solvency that they run. This note makes some related but simpler observations about the hedging properties of a deposit franchise. In particular, depending on how deposit rates behave, longer-term assets may not be a good hedge for a deposit franchise even if the depositors do not run, at least from the perspective of insulating the economic value of equity (EVE) of the bank from interest rate shocks. Moreover, a hedging strategy designed to minimize exposure of a bank's net interest margin to interest rate shocks is ill-suited for hedging the bank's EVE to those same shocks. Lastly, if deposit rates behave in accordance with the dynamics described in Dionis-Nelson (2022), which takes into account the nonlinearities caused by the zero lower bound, not only is a deposit franchise a poor hedge for longer-term assets, the franchise needs to be augmented with longer-term debt to insulate the bank from an interest rate shock similar to what happened in 2022-23.

It is worth emphasizing that these results depend critically on the infinite horizon perspective and the assumption that the portfolio of the bank is unchanged over time – in particular, that there are no net deposit inflows or outflows. These strong assumptions allow for significant simplification and isolate the issue of interest rate risk, but they also depart from standard risk analysis.

The economic value of equity of a bank is the present discounted value of its expected future profits. It is also, simplifying a bit, the fair (or market) value of its assets minus the fair (or market) value of its liabilities. Many banks report their estimate of the sensitivity of EVE to a shift in interest rates – most commonly a fixed percentage-point increase in current and future interest rates – as a measure of how well hedged the bank is to an interest rate shock. A change in expected future interest rates affects a bank's future profits for two reasons. First, when interest rates change, the yields on the banks' assets – loans and securities – change, as do the yields on its liabilities – deposits, other short-term borrowings and long-term debt. Second, a change in expected future interest rates also affects the discount factors used to calculate the present discounted value of future profits. If expected interest rates go up, the discount factors go down: the same expected future profit has a lower present value.

Ignoring risk premia, the effect of the interest rate shock on the yields and on the discount factors exactly cancels out for a short-term asset or liability that is perpetually rolled over or a term but variable-rate instrument. As a result, the present discounted value of a portfolio of overnight fed funds loans or reverse repos is unchanged by an interest rate shock.[1] Similarly, the present discounted value of perpetually rolled over fed funds borrowing or repo borrowing is also unchanged by an interest rate shock.

[1] See mathematical appendix.

For the same reason, if the rate the bank pays on its deposits is a constant β ("beta") times the fed funds rate and there are no inflows or outflows, then the value of the deposit franchise is unchanged by an interest rate shock. Specifically, the fair value of the deposits is just β times the face value of the deposits.[2]

By contrast, a term, fixed-rate instrument, such as a 30-year mortgage or long-term bond, has a value that is highly sensitive to an interest rate shock. Because the future payments on the instruments are fixed, the fair value goes up and down with changes in the discount factor. An expositionally convenient long-term fixed-rate instrument is a consol, an asset or liability with no maturity that makes a fixed periodic interest payment forever.[3] The fair value of a consol is just its interest payment times the sum of the discount factors. If interest rates go up, the discount factors go down, and therefore so does the value of the consol. In the special case when the interest rate is expected to be constant forever, the fair value of the consol is just 1 over the interest rate, a result that can be especially helpful for intuition.

From an EVE perspective, a bank whose profits are a simple multiple of the short-term rate is perfectly hedged against an interest rate shock. For example, consider a bank with a leverage ratio of 10 percent that invests $100 in fed funds sold financed with $90 in fed funds purchased. The bank's profits are $10 times the fed funds rate, which is just a simple multiple of the fed funds rate. It has an EVE of $10 and that value is unchanged in reaction to any interest rate shock. Note that $10 is also the fair value of the assets minus the fair value of the liabilities. Similarly, if the same bank is financed instead with $90 in deposits with deposit rates β times the fed funds rate, its profits would be $100 times the fed funds rate minus $90 times β times the fed funds rate, which is also just a simple multiple of the fed funds rate. Its EVE would be $100 - β $90, and it would be unchanged in response to an interest rate shock.

At the other end of the spectrum, from an EVE perspective, a bank whose profits are insensitive to interest rates is *unhedged* against interest rate risk. Suppose the fed funds rate is now and is expected forever to be r. If the same bank invested in $100 of consols with yield r funded with $90 in consols with interest rate r, its profits would be $10 * r and its EVE would be $10. The bank's NIM would be 0.1 r. Now suppose interest rates doubled to r' = 2r. The bank's profits and NIM would be unchanged, but its EVE would fall to $5, which equals its constant earnings of $10 r divided by the new interest rate r'.

The existence of fixed costs per deposit or a deposit rate that is a fixed spread under the risk-free rate adds a fixed component to bank profits that needs to be hedged by adding fixed-rate assets or liabilities to leave the bank hedged against interest rate risk from an EVE perspective. Suppose the bank is again funded with $90 in deposits but there is a cost c per dollar of deposit arising from the provision of payment services to the depositor. In that case, the expenses of the bank would be $90 (βr + c). For the profits of the bank to be a simple multiple of r, the bank will need to invest in consols generating interest income equal to $90 c to cancel out the fixed expense from the deposits. Let the consol rate be y. The bank will need to invest $90 (c/y) in consols and $100 - $90 (c/y) in fed funds sold to be hedged against interest rate risk. In that case, the bank's profits are ($100 - $90 (c/y) - $90β) r, which is a fixed multiple of r. Its EVE is ($100 - $90 (c/y) - $90β) which does not change in response to changes in r, so it is perfectly hedged against interest rate shocks.

The implications of the deposit rate being a fixed spread below the fed funds rate are similar, but hedging requires that the bank fund itself with long-term liabilities rather than invest in them. Suppose the deposit rate is just r – x and the bank funds itself with D in deposits and invests in $100 of fed funds sold. In that case, the interest expense from deposits is D(r-x). For the bank to be hedged, its interest expense needs to be a simple multiple of the fed funds rate, which it can accomplish by funding itself with consols whose interest expense equals Dx. Since the

[2] See mathematical appendix.

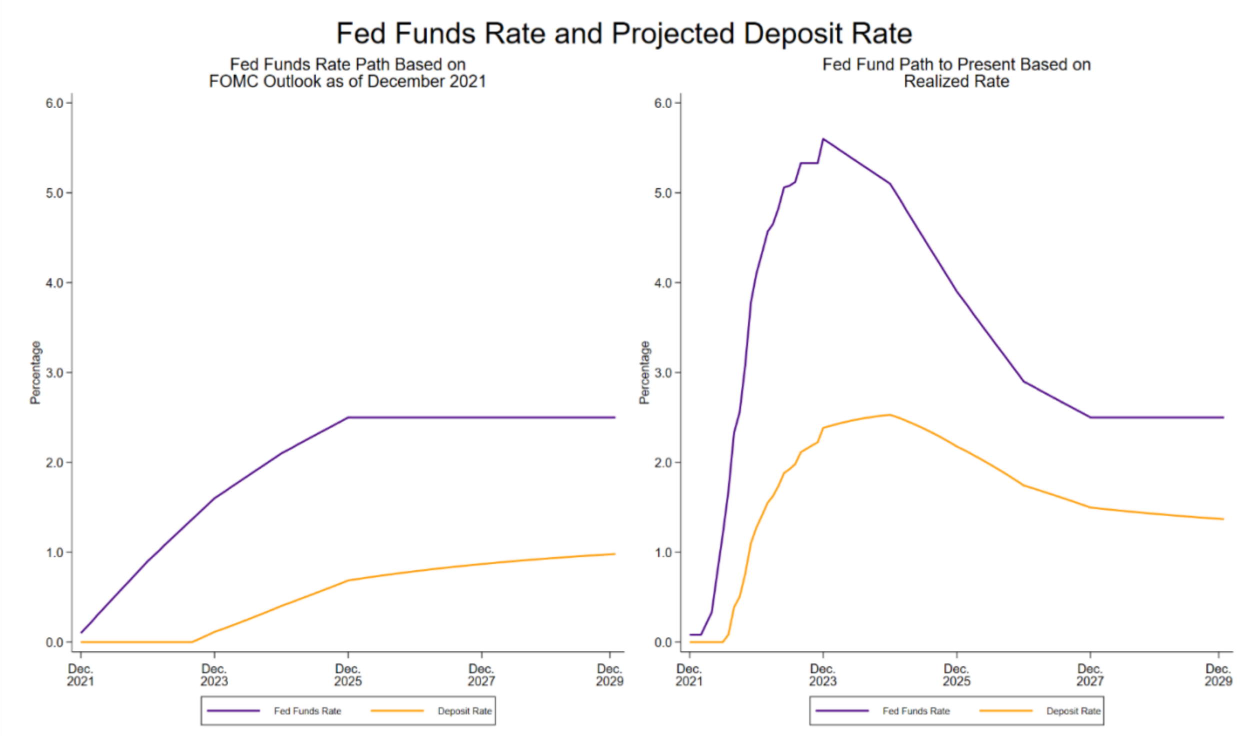
[3] Consols have been issued by the Bank of England and by the U.S. government in the past, but all have been redeemed. They are beloved by finance professors everywhere because if the short-term interest rate is a constant rate r, the value of the consol is simply its coupon divided by r.

interest expense from the consols is Cy, where C is the amount of consols, the bank needs to choose C so that C + D = $90 and Cy = Dx. The solution is C = $90/(1 + (y/x)) and D = $90 ((y/x)/(1+(y/x)).

In reality, there will be both costs associated with providing deposit services and the deposit rate will be a spread below the fed funds rate. Given the competition for deposits, the spread of the deposit rate under the fed funds rate compensates for the cost of providing the deposit. There will also be fixed costs or income associated with other activities of the bank, not just costs or profits associated with the deposit franchise, which would also require hedging.

Lastly, consider the more complicated process for deposit rates estimated in Dionis-Nelson (2022). They estimate an error correction model for the deposit rate that takes into account the nonlinearities caused by the zero lower bound – in particular, they assume that the deposit rate will not go below zero. They estimate that in equilibrium the deposit rate is 1.34 percentage points below the fed funds rate, and that in each month the deposit rate moves by about half the change in the fed funds and also closes 2 percent of the gap between its level and the equilibrium deposit rate.

Consider a bank that is funded with a $90 deposit franchise with a deposit rate that follows the DN nonlinear process that invests $100 in fed funds sold and consols. What fraction of assets should be invested in consols for the bank to be hedged against an interest rate shock as of the end of 2021? The outlook for the fed funds rate is assumed to match the median FOMC participant's outlook at the December 2021 meeting. The expected path for the fed funds rate and the expected deposit rate is shown in the left panel of the exhibit.



To complete the specification, assume further that there are costs per deposit equal to half the equilibrium spread of the fed funds rate over the deposit rate (77 basis points), and that the consol rate equals the rate that sets the market value of the consol equal to its par value under the starting outlook for the fed funds rate, which is 3.35 percent.

First, consider a standard interest rate shock in which the path is expected to be 100 basis points higher beginning immediately and forever. In that case, the share of consols for which the shock leaves the bank's EVE unchanged is -11.6 percent. That is, the bank borrows an additional $11.60 in dollars and invests $111.60 in fed funds, funded with $10 equity, $90 deposits and $11.6 in consols. Not only is the deposit franchise not a hedge against interest rate shocks, its value rises rather than falls when interest rates rise, so the bank needs to add a liability whose value falls by borrowing using consols to be hedged.

Second, consider a shock that replicates the actual interest rate path through December 2023 and the updated FOMC outlook thereafter (the right panel of the exhibit). In that case, the bank is fully hedged when it borrows $10.80 in consols and invests in additional fed funds sold, quite similar to the situation where there is a 100-basis-point shock.

In sum, two somewhat surprising results are obtained. First, if the deposit rate is β times the fed funds rate, there is no interest rate risk in the deposit franchise from an EVE perspective, so the bank does not need to invest in any longer-term assets to hedge its deposit franchise. Second, if a bank structures its assets and liabilities so that its NIM is constant, it is not hedged against interest rate risk from an EVE perspective. In addition, fixed costs or interest rate spreads may require the bank to invest in, or fund itself with, fixed-rate assets to be well-hedged. Lastly, if deposit rates follow a more complicated process in which deposit rates adjust slowly and that allows for the zero lower bound, the deposit franchise has negative duration requiring the bank to fund itself with longer-term fixed-rate liabilities to be well hedged. These counterintuitive results are based on the strong simplifying assumptions that the bank's portfolio is unchanged and infinitely lived, but those assumptions are a fairly common starting place for building economic intuition.

Mathematical Appendix

The fair value of a portfolio of fed funds loans is the par value.

Let r_t be the one-period fed funds rate and the discount rate used to calculate present value. Let F be the amount invested. In the next period, the portfolio pays off principal and interest $(1+ r_t)F$. The one period discount factor is $1/(1+r_t)$ so the present value of the payoff next period and therefore the fair value of the portfolio is $(1+ r_t)F./(1+r_t)$ which equals F. For N periods, assume the interest payment is paid out and the principal is rolled over. The interest payment in period t+k is simply $r_{t+k}F$. The discount factor for period t+k is $(1+r_t)(1+r_{t+1})\ldots(1+r_{t+k}) = \delta_{t+k}$. The fair value of the portfolio is therefore:

$$E_t\left(\frac{r_tF}{(1+r_t)}+\frac{r_{t+1}F}{(1+r_t)(1+r_{t+1})}\cdots\frac{(1+r_{t+N})F}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N})}\right)$$
$$= F\,E_t\left(\frac{r_t}{(1+r_t)}+\frac{r_{t+1}}{(1+r_t)(1+r_{t+1})}\cdots\frac{r_{t+N-1}}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N-1})}+\frac{1}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N-1})}\right)$$
$$= F\,E_t\left(\frac{r_t}{(1+r_t)}+\frac{r_{t+1}}{(1+r_t)(1+r_{t+1})}\cdots\frac{r_{t+N-2}}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N-2})}+\frac{1}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N-2})}\right) = F$$

The fair value of a deposit franchise with D deposits, no net inflows or outflows, and deposit rate βr_t is βD

Consider the above proof, but with interest payments βr_t instead of r_t and for a deposit franchise that is infinitely lived. In that case, applying the proof above, the fair value of the deposit franchise is

$$\lim_{N\to\infty} E_t\left(\frac{\beta r_tD}{(1+r_t)}+\frac{\beta r_{t+1}D}{(1+r_t)(1+r_{t+1})}\ldots+\frac{\beta(1+r_{t+N})D}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N})}-\frac{\beta D}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N})}\right)$$
$$= \beta D - \lim_{N\to\infty} E_t\left(\frac{\beta D}{(1+r_t)(1+r_{t+1})\ldots(1+r_{t+N})}\right) = \beta D$$

Note that the result requires that the interest rate is not zero or negative in equilibrium.