An Even Better Way To Combine Stress Test Results

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A few months ago, we wrote a post, “A better way to combine stress test results,” in which we proposed a way the Fed could combine the results of multiple stress tests. As we described, the Fed’s reliance on the results of a single projection generates incorrect measurements of risk and other undesirable consequences, including spurious volatility in capital requirements.

In this post, we present a new, simpler proposal that draws some intuition from engineering and from portfolio theory in finance. We also show that the current approach will not only lead to the government allocating credit but also could result in an increase in bank risk through reduced diversification. By contrast, the approach we propose would lower bank risk by reducing leverage, and would also leave credit allocation decisions by banks unchanged.

To quickly recap the motivation: In its annual capital stress tests, the Fed projects each bank’s capital levels under three scenarios – baseline, adverse, and severely adverse. The baseline scenario is the economic outlook. The severely adverse scenario basically replicates the Great Recession with higher severity. And the adverse scenario captures other ways banks can make losses apart from a recession, usually a rise in interest rates. The Fed requires banks to have enough capital so that the bank’s capital levels are above minimum requirements in each scenario, but almost invariably, the severely adverse scenario is the one that matters. As a result, banks subject to the Fed’s stress tests have underinvested in assets that struggle in a recession, such as small business loans, and overinvest in assets that do well, such as long-dated Treasury securities.

Ideally, the Fed would instead combine the results of different stress scenarios so that all the information is used. Such an approach would improve the measurement of bank risk as well as reduce the incidence of unintended consequences. Nonetheless, given the current uselessness of the adverse scenario, the Fed has proposed dropping it.

If bank failures were always or even usually the result of one thing going spectacularly badly, the Fed’s approach might make sense. Automobile accidents usually happen this way, with a single collision from the front, side, or rear. Crash tests are therefore designed to ensure cars can withstand any one of these impacts while keeping passengers safe, much like the Fed requires banks to maintain a minimum capital level under multiple scenarios.

Consider the case of designing a stress test for an automobile. There are a variety of possible accidents that could prove fatal. The likeliest (the auto equivalent of high unemployment) is a front-end collision. But there are also offset impacts, side impacts, rear impacts, rollovers, and flying debris, each of which could occur with a variation in severity. One could choose one of these potential scenarios, impose a high severity, and let that determine car design – so, a 60-mph front impact. But that would create a very strange looking car, and one quite exposed to the other risks. Conversely, one could impose a high severity for every possible scenario – 60 mph from every angle, with a rollover and large object striking the car. The result would be effectively a tank – the auto equivalent of a bank holding 100% equity and investing only in Treasuries – a safe vehicle serving no social purpose. Perhaps worst yet, one could vary the scenario year to year – say, a
60-mph front impact one year, followed by a 60-mph rollover the next; however, this would throw auto design into chaos. Instead, though, a rational actor might choose to include all the possible scenarios that could occur with an automobile, but at lower severity – say, a 30-mph impact from every side. Thus, car designers would include crumple zones and other safety measures to guard against all scenarios. In fact, this is basically what has happened in car stress scenario design.\(^1\)

Engineers deal with this sort of problem using “Tolerance Analysis.” When a product is made of multiple parts, and each part is subject to some variation in its manufacture, tolerance analysis estimates the potential variation in the entire product. Within tolerance analysis, “worst-case” analysis simply takes all the ways things can go wrong, assumes they do go wrong, and adds them up. In the bank capital context, worst-case analysis would dictate that all banks be funded 100 percent with capital or invest only in Treasury bills, or both.

“Statistical variation” analysis combines information about the statistical distribution of each part to determine the distribution of the variation of the combined product. For example, if the production requires stacking metal plates, then the statistical variation of the thickness of each plate is combined to give the statistical variation of the thickness of the stack. The variance of the thickness of the combined product is simply the sum of the variances of the thickness of the individual plates, and the standard deviation of the thickness is the square root of the sum of the variances. That statistic is known as RSS, or “root sum of squares”.

“Portfolio theory” in finance describes how investors choose to distribute their investments across assets to achieve specific outcomes. A canonical variant assumes investors seek portfolios that are mean-variance efficient; that is, they will choose portfolios that achieve the lowest possible risk for any specific level of return.

Using portfolio theory, we show that the Fed’s approach to implementing stress tests will lead banks to choose riskier portfolios than they would on their own. That perverse result occurs because banks underinvest in assets exposed to recession risk and end up insufficiently diversified. Moreover, the test has no impact on the bank’s choice of overall leverage.

We then show that if the Fed implemented its tests in a manner consistent with tolerance analysis, in particular, using RSS, banks would reduce leverage and expected return but would not change the proportional shares of the portfolio in different risky assets. That is, the RSS approach takes the Fed out of the business of credit allocation.

**TOLERANCE ANALYSIS AND PORTFOLIO THEORY**

Consider a bank that can invest in three assets: a riskless asset with return \( r \) and two risky assets that are independent, have a variance of 1, and are each in equal total supply. Because a portfolio invested equally in the two risky assets would correspond to the market portfolio, each of the risky assets has the same expected return as the market, \( r \). The example can be thought of as a simple world where there are many risky assets but only two sources of systemic risk.

The bank chooses portfolios that are mean-variance efficient. In particular, the bank chooses a portfolio whose expected return is equal to a target level \( r \) and has the lowest achievable variance.

The portfolio weights on the risky assets are \( \alpha_1 \) and \( \alpha_2 \) and the weight on the riskless asset is \((1-\alpha_1-\alpha_2)\). If the weight on the riskless asset is negative, the bank has levered its portfolio of risky assets by borrowing.

\(^1\) See “Stress Test Dummies: A Fundamental Problem with CCAR (and how to fix it),” Greg Baer, BPI Blog, July 16, 2018.
Because the two risky assets are independent, and each have variance 1, the variance of the portfolio is simply the sum of the squared risky asset portfolio weights.

The bank solves

\[
\min (\alpha_1^2 + \alpha_2^2) \tag{1}
\]

Subject to

\[
(1 - \alpha_1 - \alpha_2) r_f + \alpha_1 r^m + \alpha_2 r^m = r^t. \tag{2}
\]

The answer (we leave the derivation to the reader) is

\[
\alpha_1 = \alpha_2 = \frac{1}{2} \left( \frac{r^t - r_f}{r^m - r_f} \right). \tag{3}
\]

If the bank targets the market return, that is, \( r^t = r^m \), the fraction in brackets is equal to one and the two portfolio shares on the risky assets each equal one half. The bank simply holds half of its equity in the first risky assets and half in the other with no leverage and earns the market return. If the bank targets a higher return than the market return, \( r^t > r^m \), the portfolio weights on the risky assets are each greater than one half and so sum to more than one—the bank borrows at the riskless rate to get a higher return.

Regardless of the target rate or the other parameters, the bank always chooses to hold the two risky assets in equal amounts. That is, the bank always holds the market (recall that the two risky assets are in equal supply) and simply varies its leverage to get a higher or lower return.

**Stress Test**

Suppose the bank is also subject to a stress test where it is required to project its portfolio return under the assumption that the first risky asset declines by \( s \) percent. To pass the test, the bank’s capital (the amount invested) must stay above a minimum level, and the bank’s capital is currently \( x \) percent above that level.

If the bank can pass the test with the variance-minimizing portfolio then it will change nothing. If the bank cannot pass the test with that portfolio then the projected losses under the stress scenario exceed the bank’s capital buffer:

\[
(1 - \alpha_1 - \alpha_2)r_f - \alpha_1 s + \alpha_2 r^m < -x. \tag{4}
\]
That condition is met (the bank fails the test) when

\[ \alpha_1 > \left( \frac{r^t + x}{r^m + s} \right) \] (5)

If the bank fails the test, it is no longer able to choose the variance-minimizing portfolio that achieves its expected return target, so it must adjust its portfolio. We assume the bank chooses a new portfolio that has the lowest variance among portfolios that both achieve its return target and passes the stress test. In that case, the bank’s problem becomes solving two linear equations in two unknowns:

\[ (1 - \alpha_1 - \alpha_2)r^f + \alpha_1 r^m + \alpha_2 r^m = r^t \] (6)

and

\[ (1 - \alpha_1 - \alpha_2)r^f - \alpha_1 s + \alpha_2 r^m = -x. \] (7)

In this case, the bank chooses portfolio weights

\[ \alpha_1 = \left( \frac{r^t + x}{r^m + s} \right) \] (8)

And

\[ \alpha_2 = \left( \frac{r^t - r^f}{r^m - r^f} \right) - \alpha_1 \] (9)

Note that the portfolio weight on the first risky asset is set at the level (equation 5) that is the cutoff for the optimal weight where any higher and the bank can’t both choose the optimal weight and pass the test. Because we are assuming that the bank cannot pass the test at the unconstrained variance-minimizing choice of portfolio, it follows that the portfolio weight on the first risky asset—the asset subject to the stress test—is lower than in the optimal portfolio and the weight on the second risky asset is higher.

Note also that the sum of the weights on the risky assets is the same as the sum if the bank chooses the variance minimizing portfolio, and recall that the bank’s leverage is one minus that sum. Consequently, the bank does not respond to the stress test by varying its leverage. But because the portfolio weights on the two risky assets are no longer set to the variance-minimizing levels, the variance on the portfolio of risk assets is higher.
To recap, the stress test leads the bank to reduce its holdings of the stressed asset, increase its holdings of the other risky asset, and leave its leverage unchanged. The bank continues to achieve its target return, but it is riskier.

**A BETTER STRESS TEST**

A stress test can be constructed that ensures a target level of bank risk but also takes the government out of the credit allocation business. The test entails conducting two projections conditional on scenarios that each, individually, have a chance of occurring equal to the target level of risk and requiring each bank to have a larger capital buffer than the RSS of the two projections less the projected growth in capital under the baseline.

The objective of the test is to ensure that bank failure is an N-standard deviation event. That is accomplished by requiring that

\[ r^t - N \sqrt{\alpha_1^2 + \alpha_2^2} > -x. \]  

(10)

That is, if the portfolio earns its mean return minus N standard deviations, the return would still be higher than the capital buffer the bank has over its minimum requirement.

The stress test consists of forming projections conditional on each of the risky assets declining by s

\[ p_1 = (1 - \alpha_1 - \alpha_2)r^f - \alpha_1 s + \alpha_2 r^m \]  

(11)

\[ p_2 = (1 - \alpha_1 - \alpha_2)r^f + \alpha_1 r^m - \alpha_2 s \]  

(12)

The baseline projection is that the net portfolio return equals the target return (rt) defined above. In other words, if economic conditions develop as expected, the bank achieves the earnings it is targeting. Subtracting the baseline from the stress projections yields

\[ p_i - r^t = -\alpha_i (r^m + s) \]  

(13)

Recalling that the assets have variance and standard deviation of 1, an N-standard deviation decline in either asset is \( r^m - N \). Setting the asset decline assumed in the stress test to its N-standard deviation decline yields

\[ p_i - r^t = -\alpha_i N \]  

(14)
Consequently

\[ N \sqrt{\alpha_1^2 + \alpha_2^2} = \sqrt{(p_1 - r)^2 + (p_1 - r)^2} \quad (15) \]

So, to conduct the stress test, referring back to equation (10), subtract from the baseline projection the root sum of squares of the two stress projections each minus the baseline and check if the result would more than use up the banks’ capital buffer. Each stress projection is conditioned on the individual asset experiencing an N-standard deviation decline. The failure of a bank that passed the test would be an N-standard deviation event.

If a bank that achieved its return target with a variance-minimizing portfolio was unable to pass the test, it would have to settle for a lower return target to pass the test. It would achieve its return-maximizing portfolio subject to passing the test by reducing its overall leverage while maintaining its holdings of the two assets in equal proportion.

HOW TO IMPLEMENT THE TEST USING THE CURRENT FRAMEWORK

Shifting to capital stress tests, assume the “severely adverse” and “adverse” scenarios span the ways banks can go bad (or at least two ways that supervisors care about), and the baseline scenario describes the mean. Suppose further that the scenarios are calibrated to match the desired risk of bank failure—if bank capital should ensure the risk of failure is less than 0.5 percent a year, then the scenarios should each be set so the chance that anything worse would happen is 0.5 percent. To calculate the RSS, take the deviation of the projections under the adverse scenarios from the baseline, square them, add them up, and take the square root. Of course, this approach would only work if the Fed continued to include an adverse scenario. (Of course, under this approach, they would no longer be “severely adverse” versus “adverse” as each would be of equal probability and therefore severity, although one risk could very well be more consequential than the other.)

For example, suppose that a bank’s capital ratio is expected to increase 25 basis points under the baseline, fall 2.25 percent under the severely adverse, and fall 1.25 percent under the adverse. The RSS would be 2.92 percent. The bank should be required to hold that amount of capital minus the baseline, so 2.67 percent, over the minimum.

Because this approach requires, essentially, adding the projected capital declines under two stress tests together, other things equal it would increase capital requirements. To keep the stress tests calibrated about where they are, the severity of each scenario would need to be reduced. If the target severity of the overall test is the Great Recession, then such a reduction in scenario severity would be consistent with the requirement that the scenarios each have the same target severity as the overall test. As shown in Covas and Nelson (2018) and Covas (2018), the Fed’s severely adverse scenarios have been much more severe than the Great Recession; thus, to accomplish this result, the Fed would only need to construct the severely adverse scenario in conformance with its own guidance, which states that scenarios should be consistent with “severe recessions of the last 50 years.”

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